TIME SERIES ANALYSIS, MODELING AND CONTROL

SPRING 2025 COURSE PROJECT
ARMAV Based Control of Roll-to-Roll Manufacturing

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Introduction

Roll-to-roll manufacturing is a cornerstone of modern large-scale production, enabling the continuous fabrication of materials and devices on flexible substrates. By processing materials as they are unspooled from one roll and collected on another, this technique allows for high throughput, low cost, and consistent quality over large areas.

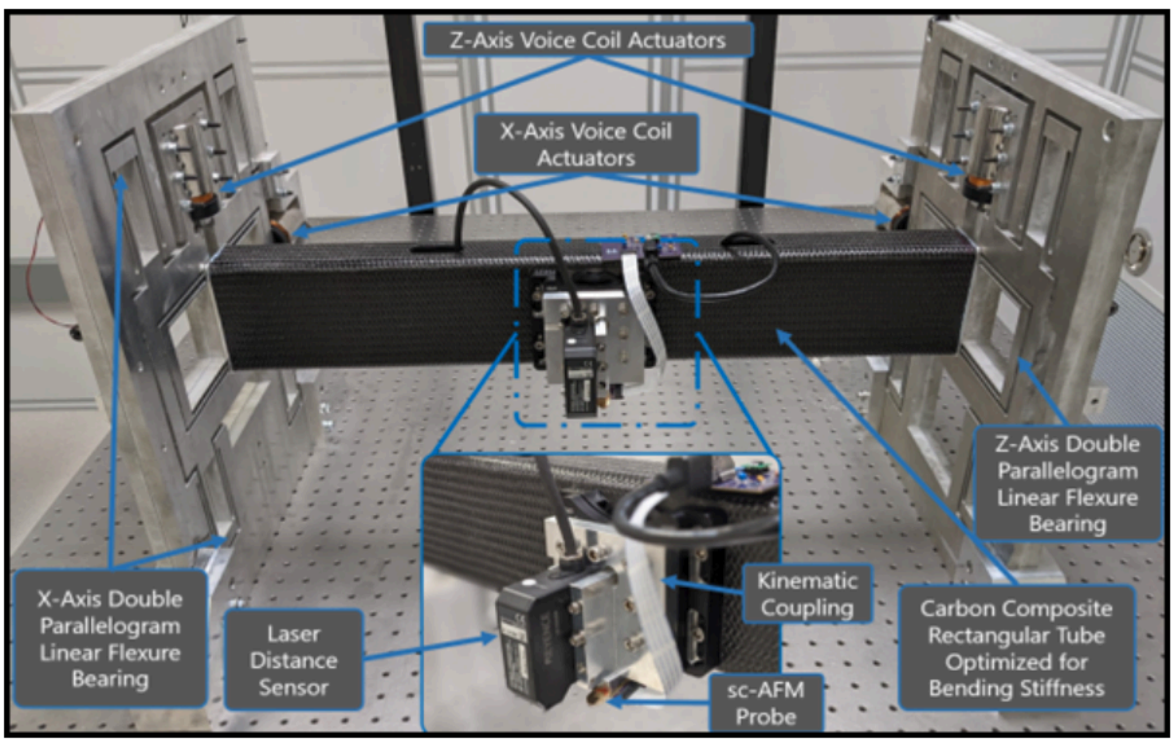
One of the primary areas where roll-to-roll manufacturing is being explored is in microelectronics. This field requires a very high level of precision and extremely small tolerances, so even minor deviations during processing can lead to defects or device failure. For this reason, minimizing controller error and implementing a robust control system is critical.

A well-designed control system with minimal errors ensures stable process conditions throughout the manufacturing line, improving device performance and yield. Ultimately, effective control systems enable Roll-to-Roll platforms to meet the stringent accuracy demands of microelectronics, allowing high-volume production without sacrificing reliability or quality.

Data Analysis

Raw Data

This dataset was taken from the previous experiments published in Dr. Barbarah Groh's Ph.D. thesis. The below image, taken from that document, depicts the system of 2 X-Axis Voice Coil Actuators controlling a flexure system. The X-Axis VCAs will be denoted as X1 and X2.



The controls system involved reducing the Multiple Input, Multiple Output system to 2 Single Input, Single Output systems, with each actuator having its own closed control system. The goal, however, was clear imaging using the sc-AFM Probe, which was dependent on the coupling of both. Reduction of error of these independently did markedly reduce error on the stage which carried the probe. The data used is on a system that already has a control loop.

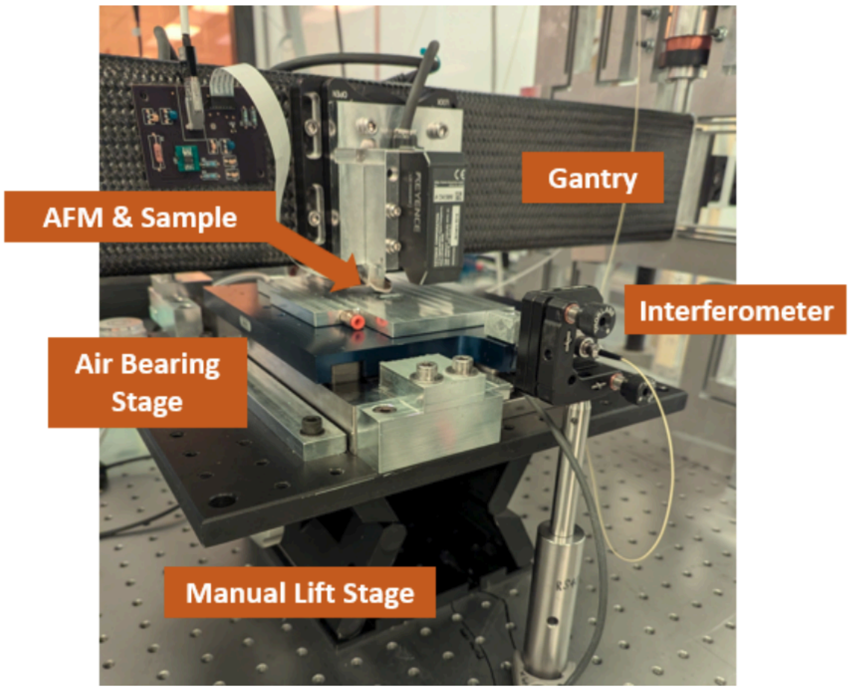


Figure 5.9: Solid substrate test setup with air bearing stage.

Data was collected via an interferometer on each X1, X2, and the Stage. The sampling rate was 1MHz, yielding a total of 48,177 data points for each part of the system. For constant velocity movement, each X1, X2, and the Stage have their target and actual position, all in unison over time. The following figure depicts this tracking.

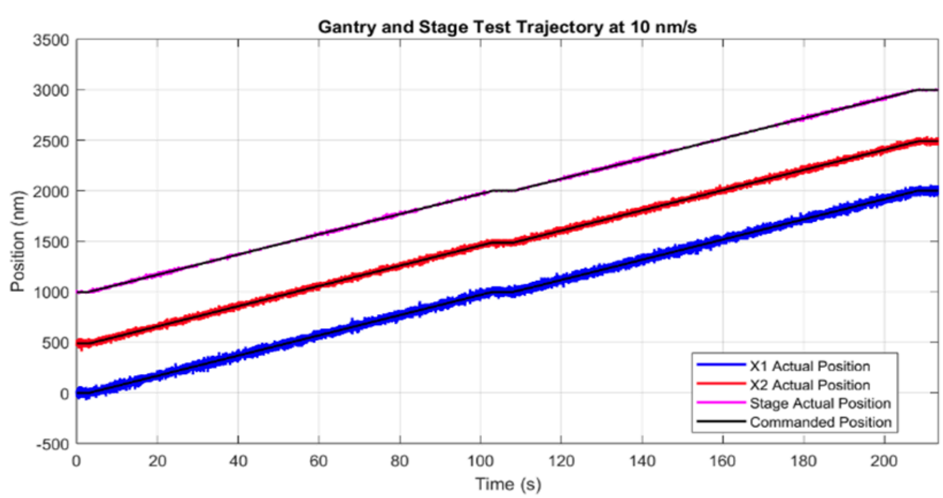
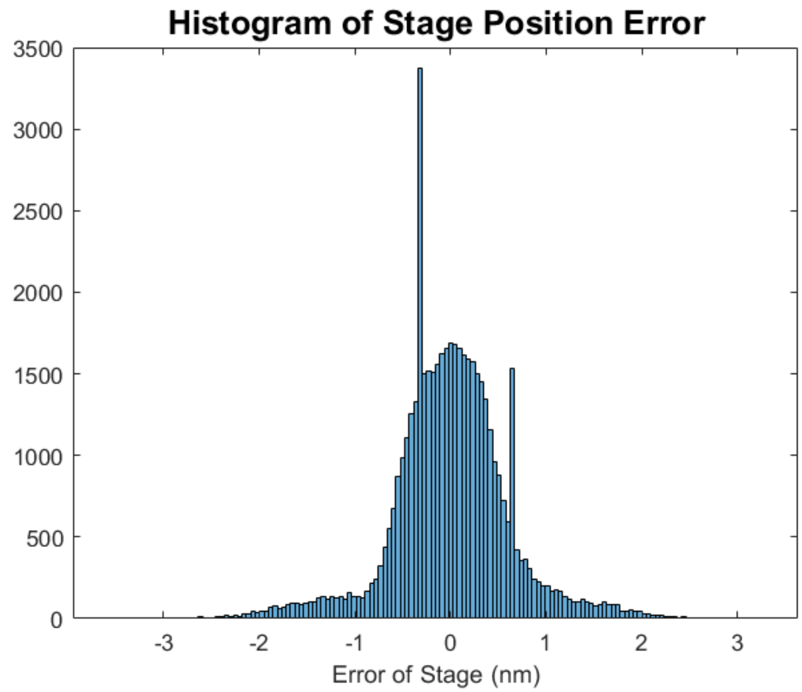


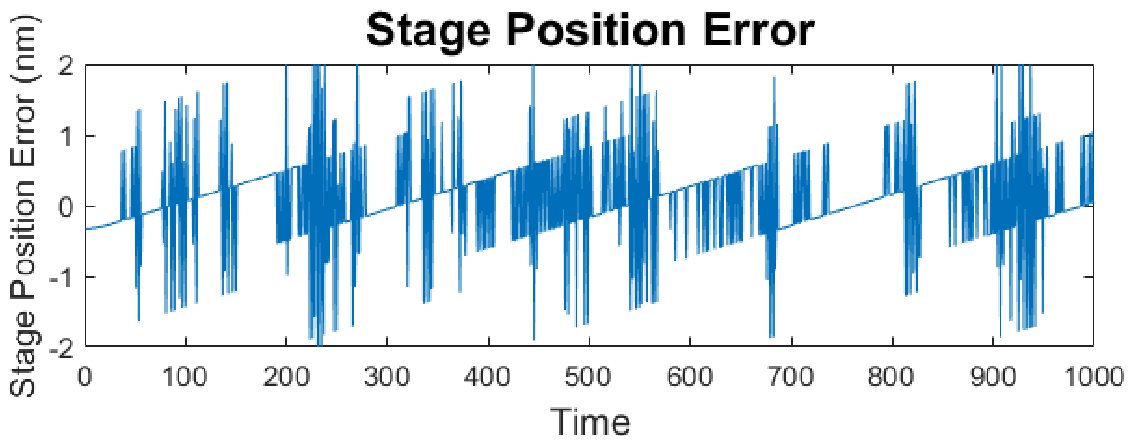
Figure 5.7: Gantry X axis trajectory tracking performance for 2 micron trajectory at 10 nm per second.

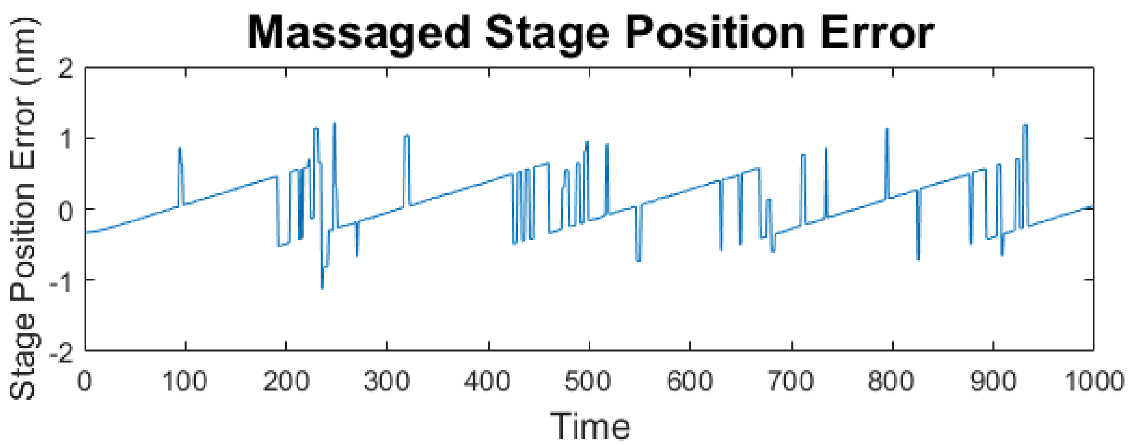
Data Cleaning

A histogram of Stage position error, calculated as the actual position subtracted from the commanded position, shows 2 strange spikes. These are specifically due to the portions of the tracking where the stage was stationary. As a result, the data was concatenated and shortened to only include portions where the stage was commanded to move. This data was further shortened to a training set containing 90% of the data and a test set containing the remaining 10%. The training set was then 40,409 elements long for each X1, X2, and the stage for actual position and target position.



A close look at the remaining stage data also shows a sharp ramping movement with noise overlain. This noise made the system quite unstable for proper ARMA analysis and resulted in a bad control system. To make the data better suited for ARMA based control, a median filter was applied. This preserves the underlying sharpness, which a mean filter would not do, while also not resulting in steps, which a decimate or resampling would do. Some error apart from the ramping, which was of a significant nature, was still present and not completely eliminated. This allowed for a more suitable system with which ARMA could be applied without removing meaning from the system. This built in drift is due to the air bearing. Any misalignment with the whole gantry with gravity results in this air bearing drift.

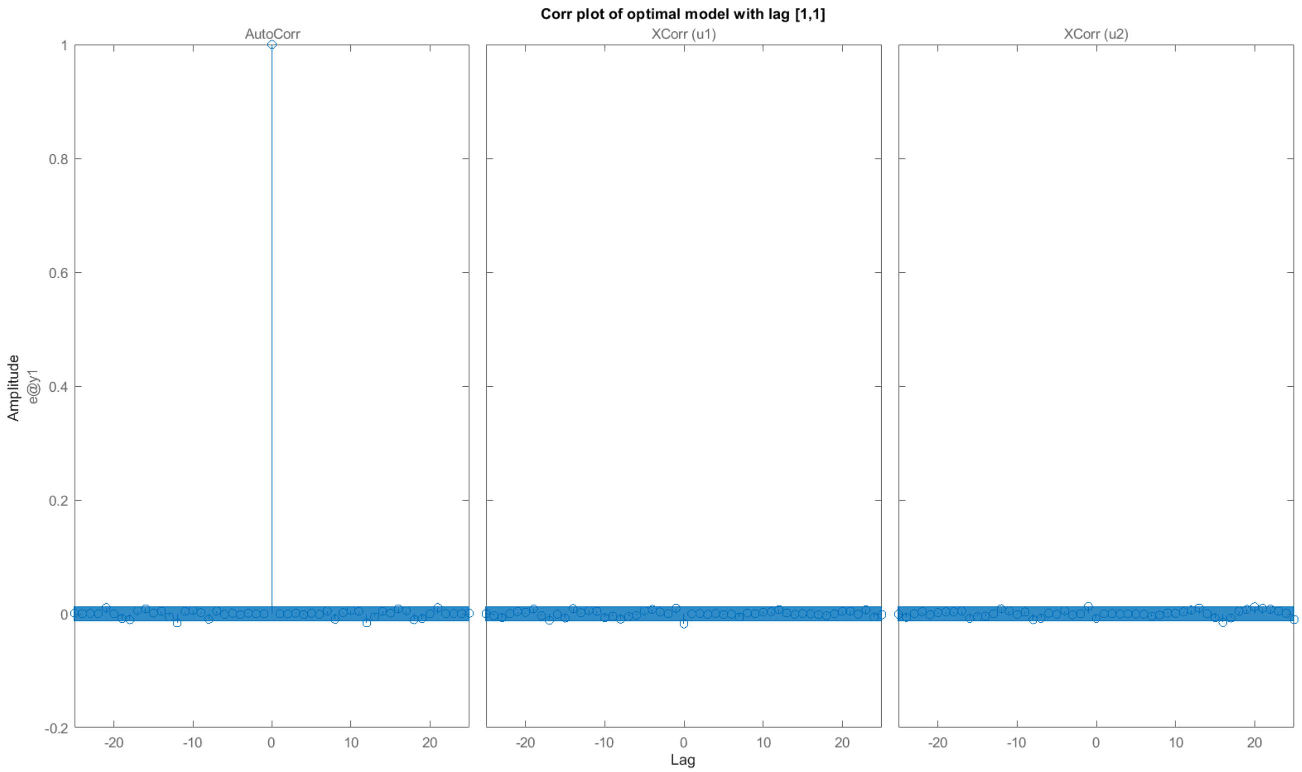




Vectorial Auto-Regressive Moving Average (ARMAV) Model

ARMAV Model Fitting

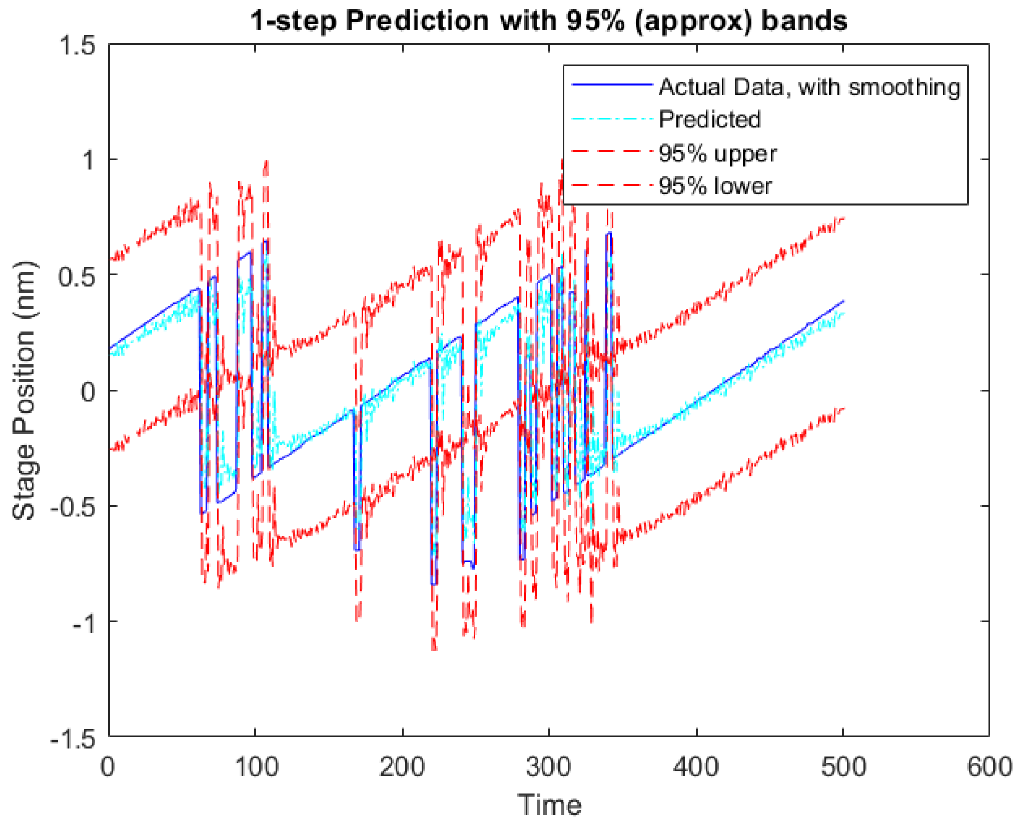
Model fitting used X1 and X2 as input channels with the Stage as an output channel. The ARMAV model then took X1 error and X2 error and previous Stage errors to predict the Stage error with lag 1. AIC fitting criterion resulted in no conclusive model, as AIC values decreased up to models of order 25, which was the most my computer could handle reasonably. F testing with critical value of 0.05, however, found a model of order 8 to be the minimally sufficient, best model. This model had 8 AR coefficients for the Stage, X1, and X2, with 7 MA coefficients. An autocorrelation plot, showing autocorrelation of stage, cross correlation of stage with X1, and cross correlation of stage with X2, shows this model to be sufficient. The mean square error of this model is 0.044. The residuals of this model were noted to be leptokurtic and sharp. While this typically indicates overfitting, this effect featured prominently in any model of order greater than 2. A model of order 2 was then tested accordingly, but was extremely inadequate as quickly verified through a correlation plot.

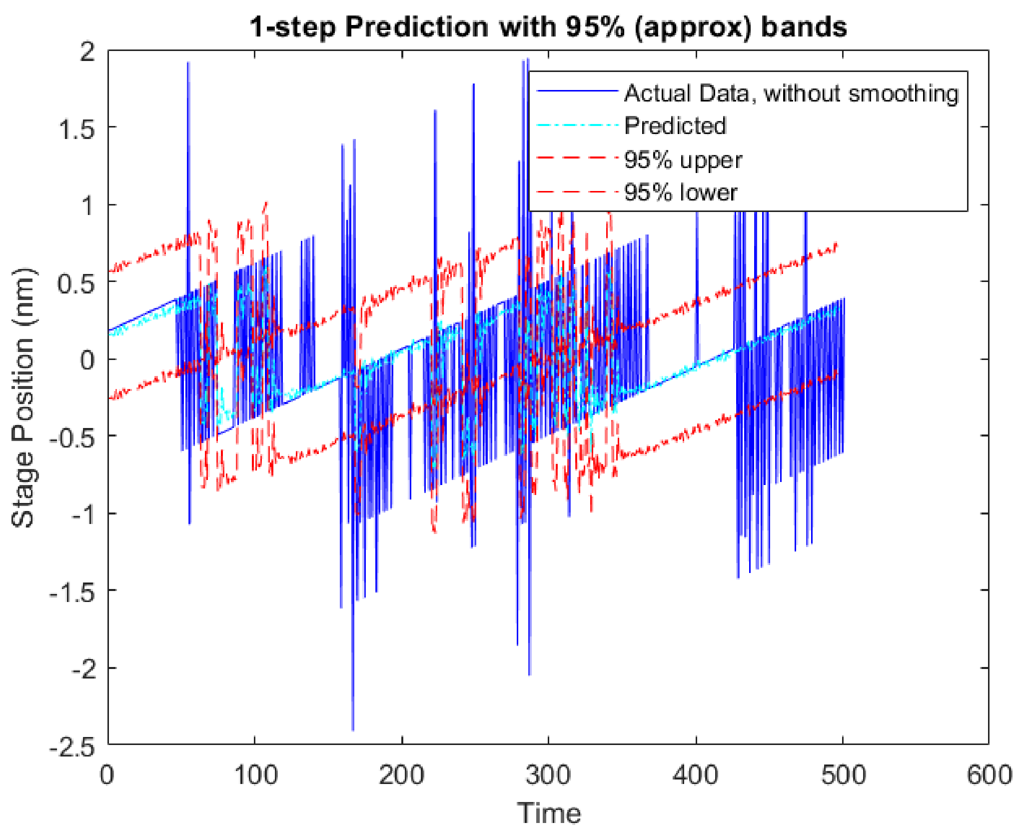


A parsimonious model of order 8, 6, 8, 7 with lag 1, meaning X1 had 6 AR coefficients instead of 8 was tested. This model failed in F-testing compared to the normal model of order 8.

ARMAV Forecasting

Another way to evaluate the validity of the ARMA model is to input measured process data and compare the model's one-step ahead predictions with the observed values. This approach provides a clear visual assessment of how closely the model follows the real system behavior. It also allows for straightforward verification of accuracy by checking whether the true data lies within the 95% confidence intervals of the predicted response. The below plots show such forecasting on elements in the testing set, first on elements with the median smoothing already applied, then on the time series without median smoothing. Considering the model was fit to the time series with median smoothing, it follows and tracks it much more closely. Unfortunately, the actual data, when not smoothed, often goes outside the 95% confidence bounds.


1-step Prediction with 95% (approx) bands
Actual Data, with smoothing
Predicted
95% upper
95% lower
Stage Position (nm)
Time
1.5
1
0.5
0
-0.5
-1
-1.5
0
100
200
300
400
500
600


1-step Prediction with 95% (approx) bands
Actual Data, without smoothing
Predicted
95% upper
95% lower
Stage Position (nm)
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
100
200
300
400
500
600

ARMAV Control Law

By setting our future predicted value to 0, we are able to derive the control law. As this is being treated as a multiple input, single output system, with 2 free variables, we need another parameter. For this, we minimize the squared sum of inputs as a stand in for energy.
The control law then is derived by setting y at n+1 to 0 and solving for u1,n and u2,n.

$$
\begin{aligned}
u_{1,n-1} + (0.00421974103489968)\, u_{2,n-1} = {} & \frac{0.095519090013101\, y_n - 0.566859455713717\, y_{n-1} - 0.797323898244570\, y_{n-2} + 0.481706061561506\, y_{n-3} + 0.061932480683540\, y_{n-4} - 0.099306189552669\, y_{n-5}}{0.00392948054299636} \\
& + \frac{-0.201562470639706\, y_{n-6} + 0.108382410323878\, y_{n-7} - 0.00392948054299636\, u_{1,n-2} + 0.00280685317788218\, u_{1,n-3}}{0.00392948054299636} \\
& + \frac{+0.00572774743677609\, u_{1,n-4} + 0.00100643327230490\, u_{1,n-5} - 0.00373474036118763\, u_{1,n-6} + 0.00147189580333855\, u_{1,n-7}}{0.00392948054299636} \\
& + \frac{- - 0.0105556640757357\, u_{2,n-2} + 0.00211917193411984\, u_{2,n-3} + 0.00705657295676815\, u_{2,n-4}}{0.00392948054299636} \\
& + \frac{-0.00492120779763902\, u_{2,n-5} - 0.00693421084290974\, u_{2,n-6} + 0.00136983722215491\, u_{2,n-7} - 0.000198598423400091\, u_{2,n-8}}{0.00392948054299636}
\end{aligned}
$$

With the additional constraint of:

$$\arg \min_{u_{n,1},\, u_{n,2}} \sqrt{u_{n,1}^2 + u_{n,2}^2}$$

Due to lag 1, on our median filtered data, our variance is just our noise term. We can use this to find our variance after control of our median filtered system and our control efficiency. Note that this is not for the real system, but for the median filtered system.

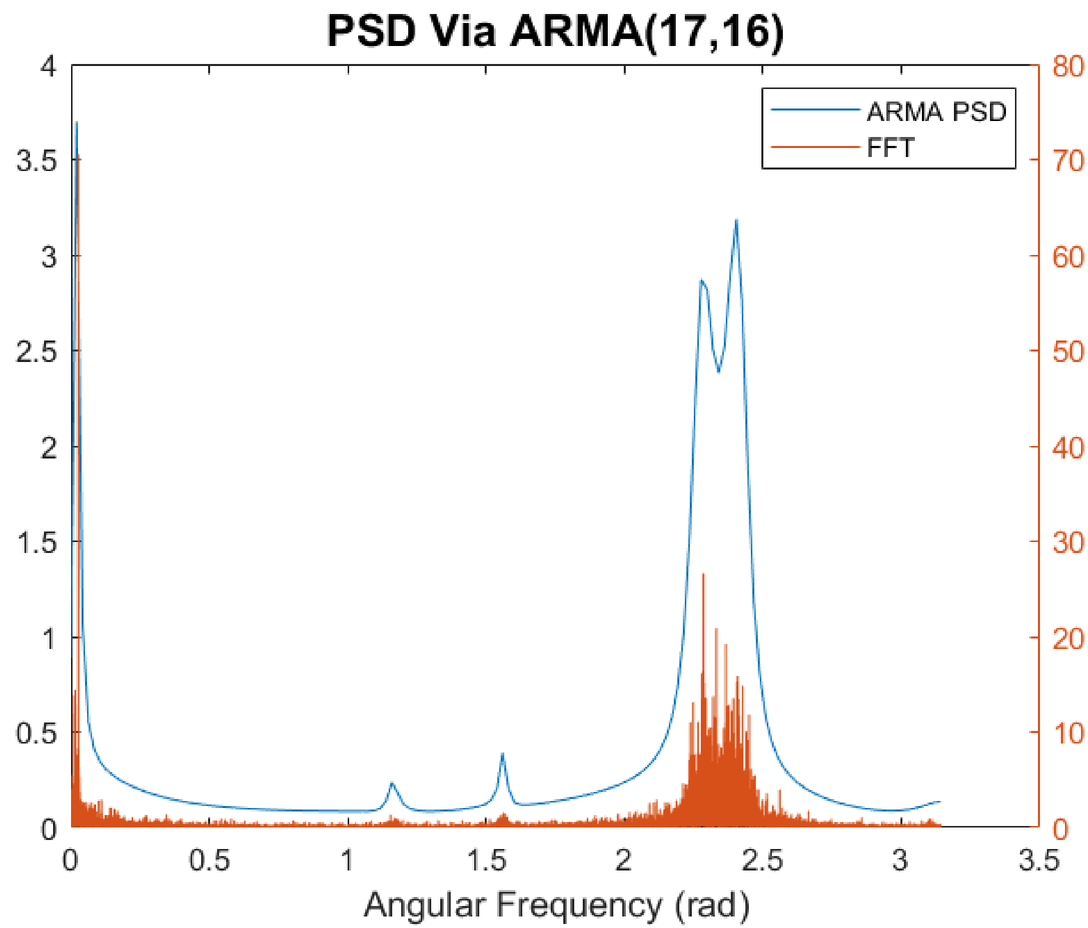
$$\text{Model after control:} \quad X_3^t = a_3^t$$

$$\text{Variance after control:} \quad \mathrm{Var}[X_3^t] = \mathrm{Var}[a_3^t] = 0.0441$$

$$\text{Control Efficiency:} \quad \frac{0.3881 - 0.0441}{0.3881} = 0.8864$$

ARMA Frequency Analysis

An ARMA model was fitted using the postulateARMA.m code provided from class. This was done on the normal stage error data, meaning it didn't include the median filter. This produced a 17, 16 ARMA model. The Power Spectral Density was then calculated via the ARMA model. This lined up closely with an FFT, although the ARMA PSD had 2 overlapping peaks that were less clearly differentiated in the FFT. When normalized to Hertz, the peaks roughly align with the 20 Hz natural frequency of the system. The low frequency spike is most likely the sharp drift of the system mentioned earlier.



Conclusion

This study shows that an ARMAV model can effectively capture the coupled error dynamics of a roll-to-roll positioning system that cannot be described by independent SISO models. Substantial data cleaning and median filtering was required, but resulted in a low mean square error and satisfactory correlation behavior. The derived ARMAV-based control law illustrates how predictive models can be used to minimize future stage error while constraining control effort. Frequency-domain analysis further validated the model by correctly identifying the system's dominant natural frequency and low-frequency drift. Overall, ARMAV modeling provides a useful framework for analysis and control of precision roll-to-roll systems, with clear potential for extension to full real-time experimental validation. The future work for this project would be implementing it on the physical system.